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$\mathcal{N} = 4$ Chern-Simons theories with auxiliary vector multiplets

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Abstract: We investigate a class of quiver-type Chern-Simons gauge theories with some Chern-Simons couplings vanishing. The vanishing of the couplings means that the corresponding vector fields are auxiliary fields. We show that these theories possess $\mathcal{N} = 4$ supersymmetry by writing down the actions and the supersymmetry transformation in terms of component fields in manifestly Spin(4) covariant form.

Keywords: Extended Supersymmetry, Chern-Simons Theories, M-Theory.

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Contents

1. Introduction

Recently, Bagger, Lambert [1 − 3], and Gusstavson [4, 5] proposed a new field theory model as a promising candidate for the theory describing multiple M2-branes. This model (BLG model) is based on Lie 3-algebras, and can also be regarded as a special class of Chern-Simons gauge theories [6, 7] with $\mathcal{N} = 8$ supersymmetry.

Until quite recent, the largest known supersymmetry of interacting Chern-Simons theories had been $\mathcal{N} = 3$. This is because supersymmetric completion of Chern-Simons terms include bi-linear terms of superpartners of gauge fields which break R-symmetry down to SO(3) (or Spin(3) when hyper multiplets are present). See [8] for detailed analysis of $\mathcal{N} = 2, 3$ superconformal Chern-Simons theories.

This symmetry breaking is, however, not necessarily physical. If there were Yang-Mills kinetic terms for the vector multiplets, the bi-linear terms would determine the masses of propagating fermions and the symmetry breaking could be seen as non-degeneracy of the masses. Then, the symmetry breaking would be physical. On the other hand, if the Yang-Mills kinetic terms are absent, as theories we investigate in this paper, the situation changes. In such a case superpartners of gauge fields become non-dynamical auxiliary fields, and there is a possibility that the R-symmetry enhances when these auxiliary fields are integrated out. The $\mathcal{N} = 8$ supersymmetry of the BLG model is a special case of such symmetry enhancement. The BLG model is very restricted, and if we require the

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algebra is finite dimensional and has positive definite metric, the only possible gauge group is SO(4) [9, 10]. (The positivity of the metric is not indispensable for the consistency of the theory. See [11 – 15].)

In the case of $\mathcal{N} < 8$, we have larger variety of theories. Gaiotto and Witten [16] showed that the supersymmetry can be enhanced to $\mathcal{N} = 4$ in a class of Chern-Simons theories with product gauge groups $\mathrm{U}(N) \times \mathrm{U}(N')$ and $\mathrm{Sp}(N) \times \mathrm{SO}(N')$. This is generalized in [17] to quiver type gauge theories by introducing twisted hypermultiplets. They construct $\mathcal{N} = 4$ Chern-Simons theories described by linear and circular quiver diagrams. A $\mathrm{U}(N) \times \mathrm{U}(N)$ Chern-Simons theory with $\mathcal{N} = 6$ supersymmetry is also proposed in [18]. For recent progress in $\mathcal{N} \geq 4$ Chern-Simons theories, see also [19 – 47].

In this paper we investugate a class of $\mathcal{N} = 4$ Chern-Simons theories. The model is described by a circular quiver diagram with circumference n. Namely, gauge group is $\prod_{I=1}^{n} \mathrm{U}(N_I)$, and there are n hypermultiplets belonging to bi-fundamental representations. The action of this model is

$$S = S_{\mathrm{CS}} + S_{\mathrm{hyper}}, \tag{1.1}$$

where S_{CS} and S_{hyper} are given in terms of $\mathcal{N} = 2$ superfields by

$$\begin{aligned} S_{\mathrm{CS}} = \sum_{I=1}^{n} k_I \mathrm{tr} \Big[\int d^3x d^4\theta \left(-\frac{i}{2} \int_0^1 dt (\overline{D}_\alpha V_I) e^{-2tV_I} (D^\alpha e^{2tV_I}) \right) \\ + \left(-\frac{i}{2} \int d^3x d^2\theta \Phi_I^2 + \mathrm{c.c.} \right) \Big], \end{aligned} \tag{1.2}$$

and

$$\begin{aligned} S_{\mathrm{hyper}} = & - \sum_{I=1}^{n} \int d^3x d^4\theta \mathrm{tr} (\overline{Q}_I e^{2V_I} Q_I e^{-2V_{I+1}} + \widetilde{Q}_I e^{-2V_I} \overline{\widetilde{Q}}_I e^{2V_{I+1}}) \\ & + \sum_{I=1}^{n} \left(\int d^3x d^2\theta \sqrt{2} i \mathrm{tr} (\widetilde{Q}_I \Phi_I Q_I - Q_I \widetilde{Q}_I \Phi_{I+1}) + \mathrm{c.c.} \right). \end{aligned} \tag{1.3}$$

A brief summary of $\mathcal{N} = 2$ superfield formalism is given in appendix A. The n vector and n hyper multiplets are labeled by the same index I. $I = n + 1$ is identified with $I = 1$. V_I and Φ_I are an $\mathcal{N} = 2$ vector and an adjoint chiral superfield, respectively, and they form an $\mathcal{N} = 4$ vector multiplet. Q_I and $\widetilde{Q}_I$ are bi-fundamental chiral superfields belonging to $(\mathbf{N}_I, \overline{\mathbf{N}}_{I+1})$ and $(\overline{\mathbf{N}}_I, \mathbf{N}_{I+1})$ of $\mathrm{U}(N_I) \times \mathrm{U}(N_{I+1})$, and these form an $\mathcal{N} = 4$ hypermultiplet.

If the Chern-Simons coupling k_I of $\mathrm{U}(N_I)$ is $k_I = (-)^I k$, this theory coincides with a model proposed in [17]. We extend the model by considering more general Chern-Simons couplings

$$k_I = \frac{k}{2}(s_I - s_{I-1}), \quad s_I = \pm 1, \quad k > 0. \tag{1.4}$$

The model in [17] corresponds to the choice $s_I = (-1)^I$. We allow s_I to be ± 1 in arbitrary order. This implies that we allow some of Chern-Simons couplings to vanish. If $k_I = 0$, all the component fields of V_I and Φ_I become auxiliary fields. We call such multiplets "auxiliary vector multiplets." For distinction we call vector multiplets with $k_I \neq 0$ "dynamical vector multiplets" although they have no propagating degrees of freedom.

Chern-Simons theories with such auxiliary vector multiplets are discussed by Gaiotto and Witten in [16]. They introduce such multiplets to define non-trivial hyper-Kähler manifolds as hyper-Kähler quotients. By integrating out the auxiliary vector multiplets in our model we obtain a Chern-Simons gauge theory coupling to sigma models with hyper-Kähler target spaces. This model is similar to the model in [17], but hyper and twisted hyper multiplets in the model are replaced by non-trivial sigma models.

The purpose of this paper is to show that our model possesses Spin(4) R-symmetry and $\mathcal{N}=4$ supersymmetry. It would be possible to prove it by extending the arguments in [17] by generalizing minimally coupled matter fields to general hyper-Kähler sigma models. In this paper, however, we adopt different way of proof. We integrate out only the auxiliary fields in the hyper and dynamical vector multiplets, and leave the component fields in the auxiliary vector multiplets in the action. A good point of this treatment is that we do not have to solve the non-linear constraints imposed on the moment maps for auxiliary gauge fields. We will show in the following sections that, after integrating out the auxiliary fields in hyper and dynamical vector multiplets, the action (1.1) can be rewritten in manifestly Spin(4) invariant form. Because $\mathcal{N}=2$ supersymmetry of our model is manifest by construction, the Spin(4) invariance of the action implies that the existence of $\mathcal{N}=4$ supersymmetry.

The expression of Chern-Simons couplings k_I in (1.4) is closely related to a brane construction of the model. Our model is the low energy limit of the theory realized on a brane system in type IIB string theory. It consists of a stack of N D3-branes wrapped on $\mathbf{S}^1$ and n fivebranes intersecting with the D3-branes. We label the fivebranes by $I=1,\ldots,n$ in order of intersections with the D3-branes along $\mathbf{S}^1$. If the charge of I-th fivebrane is $(m_I,1)$, the Chern-Simons coupling of the gauge field living on the interval of the D3-branes between two intersections I and $I-1$ is given by [48, 49]

$$k_I=\frac{1}{2\pi}(m_I-m_{I-1}). \tag{1.5}$$

If there are only two types of fivebranes, the Chern-Simons couplings are given by (1.4).

The action of gauge theory realized on this brane system is $S_{\rm YM}+S_{\rm CS}+S_{\rm hyper}$ where $S_{\rm CS}$ and $S_{\rm hyper}$ are given in (1.2) and (1.3), respectively, and $S_{\rm YM}$ includes the Yang-Mills kinetic terms. It is given by

$$S_{\rm YM}=\sum_{I=1}^n\frac{1}{g_I^2}\left[\frac{1}{2}\int d^3xd^2\theta{\rm tr}W_I^2-\int d^3xd^4\theta{\rm tr}(\overline{\Phi}_Ie^{2V_I}\Phi e^{-2V_{I+1}})\right], \tag{1.6}$$

where g_I is Yang-Mills gauge couplings depending on the position of intersecting points of branes. The brane system preserves $\mathcal{N}=3$ supersymmetry, which coincides with the supersymmetry of the Yang-Mills-Chern-Simons action $S_{\rm YM}+S_{\rm CS}+S_{\rm hyper}$.

In the low energy limit, the kinetic terms in $S_{\rm YM}$ become irrelevant because the coupling constants g_I have mass dimension 1/2. The supersymmetry enhancement in this limit is strongly suggested by an analysis of moduli space. The Higgs branch of this model is studied in [24], and it is shown that the moduli space for $N_I=1$ is an orbifold in the form

$$\mathbf{C}^4/\Gamma, \tag{1.7}$$

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where Γ is a certain discrete subgroup consisting of elements of the form

$$(z_1, z_2, z_3, z_4) \to (e^{i\alpha}z_1, e^{-i\alpha}z_2, e^{i\beta}z_3, e^{-i\beta}z_4). \tag{1.8}$$

If we assume the flat metric, this orbifold preserves $\mathcal{N}=4$ supersymmetry.

This paper is organized as follows. In the next section we rewrite the actions given above in terms of component fields. It makes Spin(4) R-symmetry and $\mathcal{N}=4$ supersymmetry of Yang-Mills-matter system $S_{\rm YM}+S_{\rm hyper}$ manifest. We emphasize that these symmetries are different from those of the Chern-Simons-matter system $S_{\rm CS}+S_{\rm hyper}$. In order to distinguish the symmetries of these two systems, we denote the Spin(4) R-symmetry and $\mathcal{N}=4$ supersymmetry of the Yang-Mills-matter system by $R_{\rm YM}$ and $\mathcal{N}=4_{\rm YM}$, while we refer to those of Chern-Simons theory as $R_{\rm CS}$ and $\mathcal{N}=4_{\rm CS}$. In section 3 $\mathcal{N}=4_{\rm CS}$ supersymmetry transformation is written down in manifestly $R_{\rm CS}$ covariant form. In section 4 we prove the $R_{\rm CS}$ invariance of the action $S_{\rm CS}+S_{\rm hyper}$. section 5 is the concluding section.

2. Action in terms of component fields

In this section we rewrite the actions given in the introduction in terms of component fields. This makes $R_{\rm YM}=$ Spin(4) R-symmetry of $S_{\rm YM}$ and $S_{\rm hyper}$ and Spin(3) R-symmetry of $S_{\rm CS}$ manifest.

Let us first rewrite the Yang-Mills action $S_{\rm YM}$ in (1.6). Although this vanishes in the low-energy limit $g_I\to\infty$ and irrelevant to our model, it may be instructive to know the explicit form of this action. It is given by

$$\begin{aligned}S_{\rm YM}=\sum_{I=1}^n\frac{1}{g_I^2}\int d^3x{\rm tr}\Big[&-\frac{1}{4}F_{I\mu\nu}F_I^{\mu\nu}+\frac{i}{2}\lambda_I^{A\dot B}\gamma^\mu D_\mu\lambda_{IA\dot B}-\frac{1}{4}D_\mu\phi_I{}^{\dot A}{}_{\dot B}D^\mu\phi_I{}^{\dot B}{}_{\dot A}\\&-\frac{i}{2}\lambda_{IA\dot B}[\phi_I{}^{\dot B}{}_{\dot C},\lambda_I^{A\dot C}]+\frac{1}{4}F_I{}^A{}_BF_I{}^B{}_A+\frac{1}{16}[\phi_I{}^{\dot A}{}_{\dot B},\phi_I{}^{\dot C}{}_{\dot D}][\phi_I{}^{\dot B}{}_{\dot A},\phi_I{}^{\dot D}{}_{\dot C}]\Big].\end{aligned} \tag{2.1}$$

This includes $\mathrm{U}(N_I)$ gauge fields $F_{I\mu\nu}$, fermions $\lambda_I^{A\dot B}$, scalars $\phi_I{}^{\dot A}{}_{\dot B}$, and auxiliary fields $F_I{}^A{}_B$. All these fields belong to the adjoint representation of $\mathrm{U}(N_I)$, and satisfy the reality conditions

$$(F_{I\mu\nu})^\dagger=F_{I\mu\nu},\quad (\lambda_I^{A\dot B})^\dagger=-\lambda_{IA\dot B},\quad (\phi_I{}^{\dot A}{}_{\dot B})^\dagger=\phi_I{}^{\dot B}{}_{\dot A},\quad (F_I{}^A{}_B)^\dagger=F_I{}^B{}_A. \tag{2.2}$$

We raise and lower pairs of SU(2) indices of bi-spinors by the relation

$$\lambda_{IA\dot B}=\epsilon_{AC}\epsilon_{\dot B\dot D}\lambda_I^{C\dot D},\quad \epsilon_{12}=\epsilon^{12}=\epsilon_{\dot 1\dot 2}=\epsilon^{\dot 1\dot 2}=1. \tag{2.3}$$

ϕ_I and F_I are traceless

$$\phi_I{}^{\dot A}{}_{\dot A}=F_I{}^A{}_A=0. \tag{2.4}$$

This action possesses global $R_{\rm YM}={\rm SU}(2)_L\times{\rm SU}(2)_R$ symmetry. ${\rm SU}(2)_L$ and ${\rm SU}(2)_R$ act on undotted indices $A,B,\ldots=1,2$ and dotted ones $\dot A,\dot B,\ldots=\dot 1,\dot 2$, respectively.

The action of hypermultiplets $S_{\rm hyper}$ in (1.3) is rewritten as

$$S_{\rm hyper}=\sum_{I=1}^n\int d^3x{\rm tr}\Big[-D_\mu\overline{q}_{IA}D^\mu q_I^A-i\overline{\psi}_I^{\dot A}\gamma^\mu D_\mu\psi_{I\dot A}-F_I{}^A{}_B(\mu_I{}^B{}_A-\widetilde{\mu}_{I-1}{}^B{}_A)$$

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$v_{I\mu}$	ϕ_I	λ_I	F_I	q_I	ψ_I
$(\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{3})$	$(\mathbf{2},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{2},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$

Table 1: $R_{\rm YM} = {\rm SU}(2)_L \times {\rm SU}(2)_R$ representations of component fields in the $\mathcal{N}=4$ supersymmetric Yang-Mills-matter system are shown. (We do not include the auxiliary fields in the hypermultiplets in this table because they do not form representations of $R_{\rm YM}$. The $R_{\rm YM}$ invariance of the action becomes manifest only after integrating them out.)

$$-i\lambda_{IA\dot{B}}(j_I^{A\dot{B}} - \widetilde{j}_{I-1}^{A\dot{B}}) + i\psi_{I\dot{B}}\overline{\psi}_I^{\dot{A}}\phi_I^{\dot{B}}{}_{\dot{A}} - i\overline{\psi}_{I-1}^{\dot{A}}\psi_{I-1\dot{B}}\phi_I^{\dot{B}}{}_{\dot{A}}$$
$$-\frac{1}{2}\nu_I^A{}_A\phi_I^{\dot{B}}{}_{\dot{C}}\phi_I^{\dot{C}}{}_{\dot{B}} - \frac{1}{2}\widetilde{\nu}_{I-1A}^A\phi_I^{\dot{B}}{}_{\dot{C}}\phi_I^{\dot{C}}{}_{\dot{B}} + \overline{q}_{IA}\phi_I^{\dot{B}}{}_{\dot{C}}q_I^A\phi_{I+1\dot{B}}^{\dot{C}}\Big]. \quad (2.5)$$

This includes scalar fields q_I and fermions ψ_I. The auxiliary fields in Q_I and $\widetilde{Q}_I$ were integrated out so that the $R_{\rm YM}$ symmetry becomes manifest. We defined bi-linears

$$\nu_I^A{}_B = q_I^A\overline{q}_{IB}, \qquad \widetilde{\nu}_I^A{}_B = \overline{q}_{IB}q_I^A, \quad (2.6)$$
$$\mu_I^A{}_B = \nu_I^A{}_B - {\rm tr} = \nu_I^A{}_B - \frac{1}{2}\nu_I^C{}_C\delta_B^A, \qquad \widetilde{\mu}_I^A{}_B = \widetilde{\nu}_I^A{}_B - {\rm tr} = \widetilde{\nu}_I^A{}_B - \frac{1}{2}\widetilde{\nu}_I^C{}_C\delta_B^A, \quad (2.7)$$

and

$$j_I^{A\dot{B}} = \sqrt{2}q_I^A\overline{\psi}_I^{\dot{B}} - \sqrt{2}\epsilon^{AC}\epsilon^{\dot{B}\dot{D}}\psi_{I\dot{D}}\overline{q}_{IC}, \quad \widetilde{j}_I^{A\dot{B}} = \sqrt{2}\overline{\psi}_I^{\dot{B}}q_I^A - \sqrt{2}\epsilon^{AC}\epsilon^{\dot{B}\dot{D}}\overline{q}_{IC}\psi_{I\dot{D}}. \quad (2.8)$$

"$-$tr" used in (2.7) represents the subtraction of the trace part of two SU(2) indices. (2.7) and (2.8) are components of current multiplets coupled by the vector multiplets. Other components in the multiplets and the supersymmetry transformation of the components are given in appendix B. Indices in (2.5) are consistently contracted, and this action is manifestly $R_{\rm YM}$ invariant. The $R_{\rm YM}$ representations of component fields are summarized in table 1.

The $\mathcal{N}=4_{\rm YM}$ supersymmetry transformation is given by

$$\delta\phi_I^{\dot{A}}{}_{\dot{B}} = 2i(\xi_{C\dot{B}}\lambda_I^{C\dot{A}}) - i\delta_{\dot{B}}^{\dot{A}}(\xi_{B\dot{C}}\lambda_I^{B\dot{C}}), \quad (2.9)$$
$$\delta v_{I\mu} = -(\xi_{A\dot{B}}\gamma_\mu\lambda_I^{A\dot{B}}), \quad (2.10)$$
$$\delta\lambda_I^{A\dot{B}} = \frac{i}{2}\gamma^{\mu\nu}\xi^{A\dot{B}}F_{I\mu\nu} + \gamma^\mu\xi^{A\dot{C}}D_\mu\phi_I^{\dot{B}}{}_{\dot{C}} + F_I^A{}_C\xi^{C\dot{B}} + \frac{1}{2}[\phi_I^{\dot{B}}{}_{\dot{C}}, \phi_I^{\dot{C}}{}_{\dot{D}}]\xi^{A\dot{D}}, \quad (2.11)$$
$$\delta F_I^A{}_B = 2i(\xi_{B\dot{C}}\gamma^\mu D_\mu\lambda_I^{A\dot{C}}) - 2i(\xi_{B\dot{C}}[\phi_I^{\dot{C}}{}_{\dot{D}}, \lambda_I^{A\dot{D}}]) - {\rm tr}, \quad (2.12)$$

for vector multiplets and

$$\delta q_I^A = \sqrt{2}i(\xi^{A\dot{B}}\psi_{I\dot{B}}), \quad (2.13)$$
$$\delta\psi_{I\dot{A}} = \sqrt{2}\xi_{C\dot{B}}\phi_I^{\dot{B}}{}_{\dot{A}}q_I^C - \sqrt{2}\xi_{C\dot{B}}q_I^C\phi_{I+1\dot{A}}^{\dot{B}} + \sqrt{2}\gamma^\mu\xi_{B\dot{A}}D_\mu q_I^B, \quad (2.14)$$

for hyper multiplets. The parameter $\xi^{A\dot{B}}$ belongs to $(\mathbf{2},\mathbf{2})$ representation of $R_{\rm YM} = {\rm SU}(2)_L \times {\rm SU}(2)_R$.

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	q_I	ψ_I
$s_I = 1$	$(\mathbf{2},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$
$s_I = -1$	$(\mathbf{1},\mathbf{2})$	$(\mathbf{2},\mathbf{1})$

Table 2: $R_{\rm CS} = {\rm SU}(2)_{+1} \times {\rm SU}(2)_{-1}$ representations of component fields of hypermultiplets are shown.

The introduction of Chern-Simons terms $S_{\rm CS}$ in (1.2) breaks the supersymmetry to $\mathcal{N} = 3$. We can see this by rewriting the action in terms of component fields.

$$S_{\rm CS} = \sum_{I=1}^{n} k_I \int d^3x {\rm tr} \left[\epsilon^{\mu\nu\rho} \left(\frac{1}{2} v_{I\mu}\partial_\nu v_{I\rho} - \frac{i}{3} v_{I\mu} v_{I\nu} v_{I\rho} \right) \right. \\ \left. + \frac{1}{2}\phi_I{}^{\dot{A}}{}_{\dot{B}} F_I^B{}_A + \frac{1}{6}\phi_I{}^{\dot{A}}{}_{\dot{B}}\phi_I{}^{\dot{B}}{}_{\dot{C}}\phi_I{}^{\dot{C}}{}_{\dot{A}} + \frac{i}{2}\lambda_I^{A\dot{B}}\lambda_{IB\dot{A}} \right]. \tag{2.15}$$

In this action, some dotted indices are contracted with undotted indices, and thus $R_{\rm YM}$ is broken to its diagonal subgroup ${\rm SU}(2)_D$. The parameter $\xi^{A\dot{B}}$ is split into the singlet and the triplet of ${\rm SU}(2)_D$, and only the triplet part of the supersymmetry is preserved by the Chern-Simons action $S_{\rm CS}$.

As we mentioned in the introduction, however, it may be possible that the symmetry enhances with the decoupling of $S_{\rm YM}$ and an appropriate choice of k_I. Indeed, it is shown in [17] that if the Chern-Simons coupling is given by (1.4) with

$$s_I = (-1)^I, \tag{2.16}$$

the R-symmetry ${\rm SU}(2)_D$ enhances to ${\rm SU}(2) \times {\rm SU}(2)$. We should note that this enhanced symmetry acts on component fields in a different way from the original ${\rm SU}(2)_L \times {\rm SU}(2)_R$ symmetry. We denote the new symmetry by $R_{\rm CS} = {\rm SU}(2)_{+1} \times {\rm SU}(2)_{-1}$. In the model with (2.16), the component fields in the hyper multiplets belongs to the representation shown in table 2 [17]. A hypermultiplet (q_I, ψ_I) with $s_I = 1$ is transformed in a different way from a multiplet with $s_I = -1$. These two types of hypermultiplets with different s_I are called hyper and twisted hyper multiplet in [17]. In the following we prove $R_{\rm CS}$ invariance of our model based on the assumption that (q_I, ψ_I) are transformed in the same way even when s_I are not given by (2.16).

In order to show the enhancement of R-symmetry, we integrate out λ_I and F_I in dynamical vector multiplets. The equation of motion of F_I is

$$\frac{k_I}{2}\phi_I{}^A{}_B = \mu_I{}^A{}_B - \widetilde{\mu}_{I-1}{}^A{}_B, \tag{2.17}$$

and we can eliminate the ϕ_I component of the dynamical vector multiplet. At the same time, F_I itself disappears from the action. The equation of motion of λ_I is

$$k_I \lambda_I^{BA} = j_I^{AB} - \widetilde{j}_{I-1}^{AB}. \tag{2.18}$$

We eliminate λ_I in the dynamical vector multiplet by this equation.

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The resulting action includes the following fields

$$\begin{cases} (q_I, \psi_I) \text{ in hyper multiplets} \\ (v_{I\mu}) \text{ in dynamical vector multiplets} \\ (v_{I\mu}, \phi_I, \lambda_I, F_I) \text{ in auxiliary vector multiplets} \end{cases} \tag{2.19}$$

3. $\mathcal{N} = 4$ supersymmetry transformation

3.1 Hyper multiplets

Now let us write down the $\mathcal{N} = 4_{\rm CS}$ supersymmetry transformation. This is achieved by rewriting $\mathcal{N} = 3$ transformation in $R_{\rm CS}$ covariant form.

$\mathcal{N} = 3$ transformation is obtained from that of $\mathcal{N} = 4_{\rm YM}$ given in the previous section by neglecting the distinction between undotted and dotted indices, and make the transformation parameter ξ_{AB} symmetric with respect to the exchange of two SU(2) indices.

From this $\mathcal{N} = 3$ transformation, we can obtain $\mathcal{N} = 4_{\rm CS}$ transformation by carefully introducing distinction between $\mathrm{SU}(2)_{+1}$ and $\mathrm{SU}(2)_{-1}$ indices so that q_I and ψ_I belongs to the representations shown in table 2, and indices are contracted among the same kind of indices. We use overlined and underlined indices for $\mathrm{SU}(2)_{+1}$ and $\mathrm{SU}(2)_{-1}$, respectively. Two indices of the parameter ξ are associated with different SU(2) in $R_{\rm CS}$. We assume that the first and the second index are acted by $\mathrm{SU}(2)_{+1}$ and $\mathrm{SU}(2)_{-1}$, respectively.

Let us rewrite the transformation of q_I in (2.13) in the $R_{\rm CS}$ covariant form. The $R_{\rm CS}$ representations of q_I and ψ_I depend on s_I, and the contraction of indices in the supersymmetry transformation also depends on s_I.

$$\delta q_I^{\overline{A}} = \sqrt{2} i (\xi^{\overline{A}\underline{B}} \psi_{I\underline{B}}) \quad (s_I = +1), \quad \delta q_I^{\underline{A}} = \sqrt{2} i (\xi^{\overline{B}\underline{A}} \psi_{I\overline{B}}) \quad (s_I = -1). \tag{3.1}$$

In the left and right transformations in (3.1), SU(2) index of ψ is contracted with the second and the first index of ξ, respectively.

In general, if we have supersymmetry transformation laws for $s_I = +1$, we can always rewrite them into transformation laws for $s_I = -1$ by replacing overlined and underlined indices by underlined and overlined ones, respectively, and exchanging two indices of the parameter ξ. In the following we give only transformation laws for $s_I = +1$.

Let us consider the transformation law of $\psi_{I\underline{A}}$. The transformation (2.14) includes ϕ_I and ϕ_{I+1}, and we treat these fields in different ways depending on k_I and k_{I+1}. If $k_I = 0$ (k_{I+1}=0) we eliminate ϕ_I (ϕ_{I+1}) by using (2.17) while we leave it in the action if $k_I \neq 0$ ($k_{I+1} \neq 0$). For example, if $k_I = 0$ and $k_{I+1} \neq 0$ we leave ϕ_I in the action and eliminate ϕ_{I+1} by (2.17). From (2.14) we obtain $\mathcal{N} = 3$ transformation as

$$\delta\psi_{I\underline{A}} = \sqrt{2}\xi_{CB}\phi_I^B{}_A q_I^C + \frac{2s_I}{k}\sqrt{2}\xi_{CB} q_I^C \widetilde{\mu}_I^B{}_A - \frac{2s_I}{k}\sqrt{2}\xi_{\overline{C}\underline{B}} q_I^{\overline{C}} \mu_{I+1\underline{A}}^{\underline{B}} + \sqrt{2}\gamma^\mu \xi_{\overline{B}\underline{A}} D_\mu q_I^{\overline{B}}. \tag{3.2}$$

We put overlines and underlines to the indices in the third and fourth terms. However, it is impossible to do it consistently in the second term.

In order to resolve this problem we introduce the following shifted field.

$$\varphi_I^A{}_B = \phi_I^A{}_B - \frac{s_I}{k}(\mu_I^A{}_B + \widetilde{\mu}_{I-1B}^A). \tag{3.3}$$

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By this field redefinition we rewrite the transformation (3.2) for general k_I and k_{I+1} as

$$
\begin{aligned}
\delta\psi_{I\underline{A}} =& \sqrt{2}\gamma^\mu \xi_{\overline{B}\underline{A}} D_\mu q_I^{\overline{B}} - \frac{\sqrt{2}s_I}{k}\xi_{\overline{C}\underline{A}}(\nu_I^{\underline{D}}{}_{\overline{D}} q_I^{\overline{C}} - q_I^{\overline{C}}\widetilde{\nu}_I^{\underline{D}}{}_{\overline{D}}) \\
&+ \left(\sqrt{2}\xi_{\overline{C}\underline{B}}\varphi_I^{\underline{B}}{}_{\underline{A}} q_I^{\overline{C}}\right)_{k_I=0} - \left(\frac{2\sqrt{2}s_I}{k}\xi_{\overline{C}\underline{B}}\widetilde{\mu}_{I-1}^{\underline{B}}{}_{\underline{A}} q_I^{\overline{C}}\right)_{k_I\neq 0} \\
&- \left(\sqrt{2}\xi_{\overline{C}\underline{B}} q_I^{\overline{C}}\varphi_{I+1}^{\underline{B}}{}_{\underline{A}}\right)_{k_{I+1}=0} + \left(\frac{2\sqrt{2}s_I}{k}\xi_{\overline{C}\underline{B}} q_I^{\overline{C}}\mu_{I+1}^{\underline{B}}{}_{\underline{A}}\right)_{k_{I+1}\neq 0} \\
&+\delta'\psi_{I\overline{A}},
\end{aligned}
\tag{3.4}
$$

where $(\cdots)_{\text{condition}}$ means that it is included only when the condition is satisfied. This transformation still includes non-covariant terms and we collected them into the last term, $\delta'\psi_{I\overline{A}}$, which is given by

$$
\delta'\psi_{I\overline{A}} = -\left(\frac{\sqrt{2}s_I}{k}\xi_{CB}(\mu_I - \widetilde{\mu}_{I-1})^B{}_A q_I^C\right)_{k_I=0} - \left(\frac{\sqrt{2}s_I}{k}\xi_{CB} q_I^C(\mu_{I+1} - \widetilde{\mu}_I)^B{}_A\right)_{k_{I+1}=0}.
\tag{3.5}
$$

We will comment on this non-covariant part at the end of the next subsection. It will there be turn out that we can easily remove this unwanted part from the transformation law.

3.2 Vector multiplets

Let us write down the $\mathcal{N} = 4_{\text{CS}}$ transformation law for vector multiplets. If a vector multiplet is dynamical, it has only one component $v_{I\mu}$ as shown in (2.19), and by using (2.18) the transformation law (2.10) is rewritten as

$$
\delta v_{I\mu} = -\frac{s_I}{k}\xi_{\overline{A}\underline{B}}\gamma_\mu(j_I^{\overline{A}\underline{B}} - \widetilde{j}_{I-1}^{\underline{B}\overline{A}}).
\tag{3.6}
$$

This is R_{CS} invariant.

In an auxiliary vector multiplet, we have four component fields. In order to write manifestly R_{CS} covariant $\mathcal{N} = 4_{\text{CS}}$ transformation laws, we need to shift the fields λ_I and F_I as well as ϕ in the following way.

$$
\lambda_I'^{AB} = \lambda_I^{AB} - \frac{s_I}{2k}(j_I^{BA} + \widetilde{j}_{I-1}^{BA}),
\tag{3.7}
$$

$$
\begin{aligned}
F_I'^A{}_B =& F_I^A{}_B + \frac{s_I}{k}(K_I^A{}_B + \widetilde{K}_{I-1}^A{}_B) \\
&+\frac{s_I}{2k}[(\mu_I + \widetilde{\mu}_{I-1})^A{}_C, \varphi_I^C{}_B] - \frac{s_I}{2k}[(\mu_I + \widetilde{\mu}_{I-1})^C{}_B, \varphi_I^A{}_C],
\end{aligned}
\tag{3.8}
$$

where K_I and $\widetilde{K}_I$ in (3.8) and J_I^μ and $\widetilde{J}_I^\mu$ appearing in (3.9) below are components of current multiplets defined in appendix B. The transformation laws of $v_{I\mu}$, φ_I, and λ_I' are manifestly covariant.

$$
\delta v_{I\mu} = -\xi_{\overline{A}\underline{B}}\gamma_\mu\left(\lambda_I'^{\overline{A}\underline{B}} + \frac{s_I}{2k}(j_I^{\overline{A}\underline{B}} + \widetilde{j}_{I-1}^{\overline{A}\underline{B}})\right),
\tag{3.9}
$$

$$
\delta\varphi_I^{\underline{A}}{}_{\underline{B}} = 2i\xi_{\overline{C}\underline{B}}\lambda_I'^{\overline{C}\underline{A}} - i\delta^{\underline{A}}_{\underline{B}}\xi_{\overline{C}\underline{D}}\lambda_I'^{\overline{C}\underline{D}},
\tag{3.10}
$$

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$$\delta\lambda_I'^{\overline{A}\underline{B}} = \frac{i}{2}\gamma^{\mu\nu}\xi^{\overline{A}\underline{B}}F_{I\mu\nu} + \frac{is_I}{2k}\gamma_\mu\xi^{\overline{A}\underline{B}}(J_I^\mu + \widetilde{J}_{I-1}^\mu) + \gamma^\mu\xi^{\overline{A}\underline{C}}D_\mu\varphi_I{}^{\underline{B}}{}_{\underline{C}} + \xi^{\overline{C}\underline{B}}F_I'^{\overline{A}}{}_{\overline{C}}$$
$$+\frac{1}{2}[\varphi_I{}^{\underline{B}}{}_{\underline{C}}, \varphi_I{}^{\underline{C}}{}_{\underline{D}}]\xi^{\overline{A}\underline{D}} + \frac{1}{2k^2}[(\mu_I + \widetilde{\mu}_{I-1})^{\overline{A}}{}_{\overline{C}}, (\mu_I + \widetilde{\mu}_{I-1})^{\overline{C}}{}_{\overline{D}}]\xi^{\overline{D}\underline{B}}. \quad (3.11)$$

The transformation of $F_I'^A{}_B$ includes non-covariant terms.

$$\begin{aligned}\delta F_I'^{\overline{A}}{}_{\overline{B}} = &\ 2i\xi_{\overline{B}\underline{C}}\gamma^\mu D_\mu\lambda_I'^{\overline{A}\underline{C}} + 2i\xi_{\overline{B}\underline{C}}[\lambda_I'^{\overline{A}\underline{D}}, \varphi_I{}^{\underline{C}}{}_{\underline{D}}] \\ &+\frac{is_I}{k}[\xi_{\overline{B}\underline{C}}(j_I + \widetilde{j}_{I-1})^{\overline{A}\underline{D}}, \varphi_I{}^{\underline{C}}{}_{\underline{D}}] \\ &-\frac{2is_I}{k}[\xi_{\overline{B}\underline{D}}\lambda_I'^{\overline{C}\underline{D}} - \mathrm{tr}, (\mu_I + \widetilde{\mu}_{I-1})^{\overline{A}}{}_{\overline{C}}] \\ &+\frac{i}{k^2}[\xi_{\overline{B}\underline{D}}(j_I + \widetilde{j}_{I-1})^{\overline{C}\underline{D}} - \mathrm{tr}, (\mu_I + \widetilde{\mu}_{I-1})_I^{\overline{A}}{}_{\overline{C}}] \\ &+\delta' F_I'^{\overline{A}}{}_{\overline{B}}. \end{aligned} \quad (3.12)$$

We collected non-covariant terms into $\delta' F_I'$. It is given by

$$\delta' F_I'^{\overline{A}}{}_{\overline{B}} = \frac{\sqrt{2}is_I}{k}\xi_{CB}(q_I^C\overline{\Psi}_I^A + \overline{\Psi}_{I-1}^A q_{I-1}^C) + \frac{\sqrt{2}is_I}{k}\xi^{CA}(\Psi_{IB}\overline{q}_{IC} + \overline{q}_{I-1C}\Psi_{I-1B}), \quad (3.13)$$

where Ψ_{IA} is the left hand side of the equation of motion $\Psi_{IA} = 0$ of the fermion ψ_{IA}.

$$\Psi_{IA} = \gamma^\mu D_\mu\psi_{IA} - \phi_I{}^B{}_A\psi_{IB} + \psi_{IB}\phi_{I+1}^B{}_A + \sqrt{2}\lambda_{IBA}q_I^B - \sqrt{2}q_I^B\lambda_{I+1BA}. \quad (3.14)$$

Among the supersymmetry transformation laws written down in the previous and this subsections, $\delta\psi_I$ and $\delta F_I'$ include non-covariant parts $\delta'\psi_I$ and $\delta' F_I'$. These non-covariant terms may be simply removed from the transformation because, as is easily checked, the action $S_{\mathrm{CS}}+S_{\mathrm{hyper}}$ is in fact invariant under the non-covariant transformation δ'. Removing these terms, we obtain completely R_{CS} covariant $\mathcal{N} = 4_{\mathrm{CS}}$ supersymmetry transformation laws.

Note that the δ' transformation does not generate physical symmetry. We can easily see that if we use equations of motion (25) and $\Psi = 0$ both $\delta'\psi$ and $\delta' F'$ vanish. Thus δ' acts trivially on fields on shell, and does not have physical significance at least in the classical theory.

4. SU(2) × SU(2) invariance of the action

In this section, we prove the R_{CS} invariance of the action $S_{\mathrm{CS}} + S_{\mathrm{hyper}}$. Here we use $\mathcal{N} = 3$ notation to simplify equations. Namely, we use plain indices without dots or lines for any SU(2). It is easy to check if each term is R_{CS} invariant or not.

We first rearrange the action into the following three parts. The first part, $\widehat{S}_{\mathrm{kin}}$, includes the kinetic terms.

$$\widehat{S}_{\mathrm{kin}} = \sum_{I=1}^n \int d^3x \mathrm{tr}\Big[k_I\epsilon^{\mu\nu\rho}\left(\frac{1}{2}v_{I\mu}\partial_\nu v_{I\rho} - \frac{i}{3}v_{I\mu}v_{I\nu}v_{I\rho}\right) -D_\mu\overline{q}_{IA}D^\mu q_I^A - i\overline{\psi}_I^A\gamma^\mu D_\mu\psi_{IA}\Big]. \quad (4.1)$$

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This part is manifestly $R_{\rm CS}$ invariant. We use hats for manifestly $R_{\rm CS}$ invariant terms.

The second part, $S_{\rm pot}$, includes potential terms

$$\begin{aligned} S_{\rm pot} = \sum_I \int d^3x {\rm tr}\Big[& \frac{k_I}{2}\phi_I{}^A{}_B F_I{}^B{}_A - F_I{}^A{}_B(\mu_I{}^B{}_A - \widetilde{\mu}_{I-1}^B{}_A) \\ & -\frac{1}{2}\nu_I{}^A{}_A\phi_I{}^B{}_C\phi_I{}^C{}_B - \frac{1}{2}\widetilde{\nu}_I{}^A{}_A\phi_{I+1}^B{}_C\phi_{I+1}^C{}_B + \frac{k_I}{6}\phi_I{}^A{}_B\phi_I{}^B{}_C\phi_I{}^C{}_A \\ & +\overline{q}_{IA}\phi_I{}^B{}_C q_I^A\phi_{I+1}^C{}_B\Big]. \end{aligned} \tag{4.2}$$

This part is analyzed in section 4.1.

The rest of the action is the following part including Yukawa terms.

$$\begin{aligned} S_{\rm Yukawa} = \sum_I \int d^3x {\rm tr}\Big[& \frac{ik_I}{2}\lambda_I^{AB}\lambda_{IBA} - i\lambda_{IAB}(j_I^{AB} - \widetilde{j}_{I-1}^{AB}) \\ & +i\psi_{IB}\overline{\psi}_I^A\phi_I{}^B{}_A - i\overline{\psi}_{I-1}^A\psi_{I-1B}\phi_I{}^B{}_A\Big]. \end{aligned} \tag{4.3}$$

This part is analyzed in section 4.2.

4.1 Potential terms

We decompose the potential term by

$$S_{\rm pot} = \sum_{I=1}^n (S_{\rm pot1}^{I(k_I)} + S_{\rm pot2}^{I(k_I,k_{I+1})}), \tag{4.4}$$

where $S_{\rm pot1}^{I(k_I)}$ and $S_{\rm pot2}^{I(k_I,k_{I+1})}$ are defined by

$$\begin{aligned} S_{\rm pot1}^{I(k_I)} &= \int d^3x {\rm tr}\Big[\frac{k_I}{2}\phi_I{}^A{}_B F_I{}^B{}_A - F_I{}^A{}_B(\mu_I{}^B{}_A - \widetilde{\mu}_{I-1}^B{}_A) \\ &\qquad -\frac{1}{2}\nu_I{}^A{}_A\phi_I{}^B{}_C\phi_I{}^C{}_B - \frac{1}{2}\widetilde{\nu}_{I-1}^A{}_A\phi_I{}^B{}_C\phi_I{}^C{}_B + \frac{k_I}{6}\phi_I{}^A{}_B\phi_I{}^B{}_C\phi_I{}^C{}_A\Big], \end{aligned} \tag{4.5}$$

$$S_{\rm pot2}^{I(k_I,k_{I+1})} = \int d^3x {\rm tr}(\overline{q}_{IA}\phi_I{}^B{}_C q_I^A\phi_{I+1}^C{}_B). \tag{4.6}$$

$S_{\rm pot1}^{I(k_I)}$ includes only one ϕ_I while $S_{\rm pot2}^{I(k_I,k_{I+1})}$ includes ϕ_I and ϕ_{I+1}.

We first consider $S_{\rm pot1}^{I(k_I)}$. When $k_I \neq 0$, we eliminate ϕ_I by using (2.17). Then $S_{\rm pot1}^{I(k_I)}$ includes only scalar fields q_I, q_{I-1}, and their Hermitian conjugates.

$$\begin{aligned} S_{\rm pot1}^{I(k_I\neq 0)} &= \int d^3x {\rm tr}\left[\frac{4}{k^2}q_I^A\widetilde{\mu}_I{}^B{}_C\overline{q}_{IA}\widetilde{\mu}_{I-1}^C{}_B + \frac{4}{k^2}\overline{q}_{I-1B}\mu_{I-1}^A{}_C q_{I-1}^B\mu_I{}^C{}_A\right] \\ &\quad +\widehat{S}_{\rm pot1}^{I(k_I\neq 0)}, \end{aligned} \tag{4.7}$$

$$\begin{aligned} \widehat{S}_{\rm pot1}^{I(k_I\neq 0)} = \frac{2}{k^2}\int d^3x {\rm tr}\Big[& -\mu_I{}^A{}_B\mu_I{}^B{}_A\widetilde{\nu}_{I-1}^C{}_C - \widetilde{\mu}_{I-1}^A{}_B\widetilde{\mu}_{I-1}^B{}_A\nu_I{}^C{}_C \\ & -\nu_I{}^A{}_A\mu_I{}^B{}_C\mu_I{}^C{}_B - \widetilde{\nu}_{I-1}^A{}_A\widetilde{\mu}_{I-1}^B{}_C\widetilde{\mu}_{I-1}^C{}_B \\ & +\frac{2}{3}\mu_I{}^A{}_B\mu_I{}^B{}_C\mu_I{}^C{}_A - \frac{2}{3}\widetilde{\mu}_{I-1}^A{}_B\widetilde{\mu}_{I-1}^B{}_C\widetilde{\mu}_{I-1}^C{}_A\Big]. \end{aligned} \tag{4.8}$$

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Because we now assume $k_I \neq 0$, q_I and q_{I-1} are transformed by different SU(2) factors in $R_{\rm CS}$. Thus, if SU(2) indices of q_I and those of q_{I-1} are contracted, the term breaks the $R_{\rm CS}$ symmetry. To prove the $R_{\rm CS}$ invariance of the action, we need to show that such terms cancel among them when we sum up all terms in the action. By this reason, we separate manifestly $R_{\rm CS}$ invariant terms and denote them by $\widehat{S}^{I(k_I)}_{\rm pot1}$. In each term in $\widehat{S}^{I(k_I)}_{\rm pot1}$ indices of q_I and those of q_{I-1} are separately contracted. Contrary, in the first line of (4.7) some indices of q_I are contracted with q_{I-1}, and breaks the $R_{\rm CS}$ symmetry.

When $k_I = 0$, we rewrite the field ϕ_I and F_I by the $R_{\rm CS}$ covariant field φ_I and F'_I defined in section 3. We obtain

$$S^{I(k_I=0)}_{\rm pot1} = \int d^3x {\rm tr}\left[-\frac{2s_I}{k}\widetilde{\nu}^A_{I-1B}\widetilde{\nu}^B_{I-1C}\varphi^C_{I\ A} - \frac{2s_I}{k}\nu^A_{I\ C}\nu^B_{I\ A}\varphi^C_{I\ B}\right] + \widehat{S}^{I(k_I=0)}_{\rm pot1} + C^I, \quad (4.9)$$

where we collected $R_{\rm CS}$ invariant terms into $\widehat{S}^{I(k_I=0)}_{\rm pot1}$

$$\begin{aligned} \widehat{S}^{I(k_I=0)}_{\rm pot1} = \int d^3x {\rm tr}\Big[& -\frac{1}{2}\nu^A_{I\ A}\varphi^B_{I\ C}\varphi^C_{I\ B} - \frac{2}{k^2}\nu^A_{I\ A}\mu^B_{I\ C}\mu^C_{I\ B} \\ & -\frac{1}{2}\widetilde{\nu}^A_{I-1A}\varphi^B_{I\ C}\varphi^C_{I\ B} - \frac{2}{k^2}\widetilde{\nu}^A_{I-1A}\widetilde{\mu}^B_{I-1C}\widetilde{\mu}^C_{I-1B} \\ & -F'^A_{I\ B}(\mu_I - \widetilde{\mu}_{I-1})^B{}_A \\ & +\frac{1}{2k^2}(\nu_I + \widetilde{\nu}_{I-1})^A{}_A(\mu_I - \widetilde{\mu}_{I-1})^B{}_C(\mu_I - \widetilde{\mu}_{I-1})^C{}_B\Big], \end{aligned} \quad (4.10)$$

and C^I is defined by

$$C^I = \frac{is}{k}(\psi_{IA}\overline{\psi}^B_I + \overline{\psi}^B_{I-1}\psi_{I-1A})(\mu^A_{I\ B} - \widetilde{\mu}^A_{I-1B}). \quad (4.11)$$

It is convenient to write (4.7) and (4.9) in the unified form

$$S^{I(k_I)}_{\rm pot1} = B^{I(k_I)} + A^{I(k_I)} + \widehat{S}^{I(k_I)}_{\rm pot1} + (C^I)_{k_I=0}, \quad (4.12)$$

where $A^{I(k_I)}$ and $B^{I(k_I)}$ are defined by

$$A^{I(k_I\neq 0)} = \frac{4}{k^2}\int d^3x {\rm tr}(q^A_I\widetilde{\mu}^B_{I\ C}\overline{q}_{IA}\widetilde{\mu}^C_{I-1B}), \quad (4.13)$$

$$A^{I(k_I=0)} = \frac{s_I}{k}\int d^3x {\rm tr}(-2\nu^A_{I\ C}\nu^B_{I\ A}\varphi^C_{I\ B} + q^C_I\phi^A_{I+1B}\overline{q}_{IC}(\mu_I - \widetilde{\mu}_{I-1})^B{}_A), \quad (4.14)$$

$$B^{I(k_I\neq 0)} = \frac{4}{k^2}\int d^3x {\rm tr}(\overline{q}_{I-1B}\mu^A_{I-1C}q^B_{I-1}\mu^C_{I\ A}), \quad (4.15)$$

$$B^{I(k_I=0)} = \frac{s_I}{k}\int d^3x {\rm tr}(-2\widetilde{\nu}^A_{I-1B}\widetilde{\nu}^B_{I-1C}\varphi^C_{I\ A} - \overline{q}_{I-1C}\phi^A_{I-1B}q^C_{I-1}(\mu_I - \widetilde{\mu}_{I-1})^B{}_A). \quad (4.16)$$

Next, let us consider $S^{I(k_I,k_{I+1})}_{\rm pot2}$. This term contains ϕ_I and ϕ_{I+1}, and we need to consider four cases separately according to whether k_I and k_{I+1} are zero or not. When $k_I \neq 0$, we use (2.17) to eliminate ϕ_I, and when $k_I = 0$ we rewrite the field ϕ_I according to (3.3). We treat ϕ_{I+1} in the same way, too. The result is

$$S^{I(k_I,k_{I+1})}_{\rm pot2} = -A^{I(k_I)} - B^{I+1(k_{I+1})} + \widehat{S}^{I(k_I,k_{I+1})}_{\rm pot2}. \quad (4.17)$$

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We collected manifestly $R_{\rm CS}$ invariant terms into $\widehat{S}^{I(k_I,k_{I+1})}_{\rm pot2}$. It is given by

$$\widehat{S}^{I(k_I\neq 0,k_{I+1}\neq 0)}_{\rm pot2} = \int d^3x{\rm tr}\left[\frac{4}{k^2}\overline{q}_{IA}\mu_I^B{}_C q_I^A\widetilde{\mu}_I^C{}_B + \frac{4}{k^2}\overline{q}_{IA}\widetilde{\mu}^B_{I-1C}q_I^A\mu^C_{I+1B}\right], \quad (4.18)$$

$$\widehat{S}^{I(k_I\neq 0,k_{I+1}=0)}_{\rm pot2} = \int d^3x{\rm tr}\left[-\frac{4}{k^2}\overline{q}_{IA}\mu_I^B{}_C q_I^A\widetilde{\mu}_I^C{}_B - \frac{2s_I}{k}\overline{q}_{IA}\widetilde{\mu}^B_{I-1C}q_I^A\varphi^C_{I+1B}\right], \quad (4.19)$$

$$\widehat{S}^{I(k_I=0,k_{I+1}\neq 0)}_{\rm pot2} = \int d^3x{\rm tr}\left[-\frac{2s_I}{k}\overline{q}_{IA}\varphi_I^B{}_C q_I^A\mu^C_{I+1B} + \frac{4}{k^2}\overline{q}_{IA}\mu_I^B{}_C q_I^A\widetilde{\mu}_I^C{}_B\right], \quad (4.20)$$

$$\begin{aligned}\widehat{S}^{I(k_I=0,k_{I+1}=0)}_{\rm pot2} &= \int d^3x{\rm tr}\Big[\overline{q}_{IA}\varphi_I^B{}_C q_I^A\varphi^C_{I+1B} - \frac{4}{k^2}\overline{q}_{IA}\mu_I^B{}_C q_I^A\widetilde{\mu}_I^C{}_B \\ &\quad +\frac{1}{k^2}\overline{q}_{IC}(\mu_I-\widetilde{\mu}_{I-1})^A{}_B q_I^C(\mu_{I+1}-\widetilde{\mu})^B{}_A\Big]. \end{aligned} \quad (4.21)$$

If we sum up (4.12) and (4.17) over all I, all $A^{I(k_I)}$ and $B^{I(k_I)}$ cancel and we obtain

$$S_{\rm pot} = \sum_{I=1}^n(\widehat{S}^{I(k_I)}_{\rm pot1} + \widehat{S}^{I(k_I,k_{I+1})}_{\rm pot2}) + \sum_{k_I=0} C^I. \quad (4.22)$$

4.2 Yukawa terms

Let us consider $S_{\rm Yukawa}$ in (4.3). We decompose it as

$$S_{\rm Yukawa} = \sum_{I=1}^n S^{I(k_I)}_{\rm Yukawa}, \quad (4.23)$$

where

$$\begin{aligned}S^{I(k_I)}_{\rm Yukawa} &= \int d^3x{\rm tr}\Big[\frac{ik_I}{2}\lambda_I^{AB}\lambda_{IBA} - i\lambda_{IAB}(j_I^{AB}-\widetilde{j}^{AB}_{I-1}) \\ &\quad +i\psi_{IB}\overline{\psi}_I^A\phi_I^B{}_A - i\overline{\psi}^A_{I-1}\psi_{I-1B}\phi_I^B{}_A\Big]. \end{aligned} \quad (4.24)$$

Again we should discuss two cases with $k_I\neq 0$ and $k_I=0$ separately.

If $k_I\neq 0$, eliminating λ_I by using the equation of motion (2.18), and rewriting ϕ_I by (2.17), we obtain

$$S^{I(k_I\neq 0)}_{\rm Yukawa} = \frac{i}{k_I}(Y_{I-1}+X_I) + \widehat{S}^{I(k_I\neq 0)}_{\rm Yukawa}, \quad (4.25)$$

where we defined

$$X_I = \int d^3x{\rm tr}\left[-\frac{1}{2}j_I^{AB}j_{IBA} + 2\psi_{IB}\overline{\psi}_I^A\mu_I^B{}_A\right], \quad (4.26)$$

$$Y_I = \int d^3x{\rm tr}\left[-\frac{1}{2}\widetilde{j}_I^{AB}\widetilde{j}_{IBA} + 2\overline{\psi}_I^A\psi_{IB}\widetilde{\mu}_I^B{}_A\right], \quad (4.27)$$

and

$$\widehat{S}^{I(k_I\neq 0)}_{\rm Yukawa} = \frac{i}{k_I}\int d^3x{\rm tr}\Big[\widetilde{j}_{I-1BA}j_I^{AB} - 2\psi_{IB}\overline{\psi}_I^A\widetilde{\mu}^B_{I-1A} - 2\overline{\psi}^A_{I-1}\psi_{I-1B}\mu_I^B{}_A\Big]. \quad (4.28)$$

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When $k_I \neq 0$, q_I and ψ_I are rotated by the same SU(2) as ψ_{I-1} and q_{I-1}, respectively, and we see that terms in $\widehat{S}^{I(k_I\neq 0)}_{\text{Yukawa}}$ are manifestly R_{CS} invariant while X and Y are not. We define

$$\widehat{Z}_I = \int d^3x \text{tr}[\epsilon_{AB}\epsilon_{CD} q_I^A \overline{\psi}_I^C q_I^B \overline{\psi}_I^D - \epsilon^{AB}\epsilon^{CD} \overline{q}_{IA}\psi_{IC}\overline{q}_{IB}\psi_{ID} + \psi_{IA}\overline{\psi}_I^A q_I^B \overline{q}_{IB} - \overline{\psi}_I^A \psi_{IA}\overline{q}_{IB} q_I^B]. \tag{4.29}$$

This is manifestly R_{SC} invariant, and the following identity holds.

$$Y_I - X_I = \widehat{Z}_I. \tag{4.30}$$

By using this identity, we can rewrite the action (4.25) as

$$S^{I(k_I\neq 0)}_{\text{Yukawa}} = \frac{i}{k}\Big[-s_{I-1}X_{I-1} + s_I X_I\Big] - \frac{is_{I-1}}{k} Z_{I-1} + \widehat{S}^{I(k_I\neq 0)}_{\text{Yukawa}}, \tag{4.31}$$

where we used the relation $s_I = -s_{I-1}$, which holds when $k_I \neq 0$.

Next, let us consider $k_I = 0$ case. Rewriting ϕ_I and λ_I in the action according to (3.3) and (3.7) we obtain

$$\begin{aligned} S^{I(k_I=0)}_{\text{Yukawa}} &= \frac{is_I}{k}(-Y_{I-1} + X_I) + \widehat{S}^{I(k_I=0)}_{\text{Yukawa}} \\ &= \frac{i}{k}(-s_{I-1}X_{I-1} + s_I X_I) - \frac{is_{I-1}}{k}\widehat{Z}_{I-1} + \widehat{S}^{I(k_I=0)}_{\text{Yukawa}} - C^I, \end{aligned} \tag{4.32}$$

where C^I is defined in (4.11), and $\widehat{S}^{I(k_I=0)}_{\text{Yukawa}}$ includes R_{CS} invariant terms.

$$\widehat{S}^{I(k_I=0)}_{\text{Yukawa}} = \int d^3x \text{tr}\Big[-i\overline{\psi}^A_{I-1}\psi_{I-1B}\varphi_I^B{}_A + i\psi_{IB}\overline{\psi}_I^A\varphi_I^B{}_A - i\lambda'_{IAB}(j_I^{AB} - \widetilde{j}^{AB}_{I-1})\Big]. \tag{4.33}$$

Summing up $S^{I(k_I)}_{\text{Yukawa}}$ in (4.31) and (4.32) over all I, terms with X_I and Y_I cancel, and we obtain

$$S_{\text{Yukawa}} = \sum_{I=1}^{n}\left(-\frac{is_I}{k}\widehat{Z}_I + \widehat{S}^{I(k_I)}_{\text{Yukawa}}\right) - \sum_{k_I=0} C^I. \tag{4.34}$$

Adding (4.22) and (4.34), we obtain the manifestly R_{CS} invariant action

$$S_{\text{CS}} + S_{\text{hyper}} = \widehat{S}_{\text{kin}} + \sum_{I=1}^{n}\left(\widehat{S}^{I(k_I)}_{\text{pot1}} + \widehat{S}^{I(k_I,k_{I+1})}_{\text{pot2}} - \frac{is_I}{k}\widehat{Z}_I + \widehat{S}^{I(k_I)}_{\text{Yukawa}}\right), \tag{4.35}$$

and the proof is completed.

5. Conclusions

In this paper we investigated the Spin(4) R-symmetry and $\mathcal{N} = 4$ supersymmetry of the three-dimensional Chern-Simons-matter system described by the action $S_{\text{CS}} + S_{\text{hyper}}$, where S_{CS} and S_{hyper} are given in (1.2) and (1.3), respectively. This model consists of dynamical and auxiliary vector multiplets and bi-fundamental hypermultiplets. The dynamical vector multiplets have Chern-Simons couplings $\pm k$ while the auxiliary vector multiplets do not

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have Chern-Simons terms. (Although we call vector multiplets with non-vanishing Chern-Simons couplings "dynamical" for distinction, they do not have propagating degrees of freedom.) After integrating out auxiliary fields in the hyper and dynamical vector multiplets, our model includes (q_I, ψ_I) in the hypermultiplets, $(v_{I\mu})$ in the dynamical vector multiplets, and $(v_{I\mu}, \varphi_I, \lambda'_I, F'_I)$ in the auxiliary vector multiplets. We wrote down the $\mathcal{N} = 4$ supersymmetry transformation in terms of these component fields in manifestly Spin(4) covariant form in eqs. (3.1), (3.4), and (3.9)–(3.12). We also proved the $\mathcal{N} = 4$ invariance of the action in section 4 by rewriting it in the manifestly Spin(4) invariant form (4.35).

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A. $\mathcal{N} = 4$ multiplets and $\mathcal{N} = 2$ superfields

In this appendix we summarize our conventions for spinors and superfields. Because all we need in this paper are actions and transformation laws in terms of component fields, which are given in the main text, we here do not present detail of the superfield formalism. The purpose of this appendix is to show rough relation between components and superfields.

We use $(- + +)$ signature for the metric, and γ^μ are real 2×2 matrices satisfying

$$\eta^{\mu\nu} = \frac{1}{2}\mathrm{tr}(\gamma^\mu\gamma^\nu), \quad \epsilon^{\mu\nu\rho} = \frac{1}{2}\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho). \tag{A.1}$$

To make fermion bi-linears, we use the antisymmetric tensor $\epsilon_{\alpha\beta}$ defined by

$$\epsilon_{12} = -\epsilon_{21} = 1. \tag{A.2}$$

For example,

$$(\eta\chi) = \eta^\alpha\epsilon_{\alpha\beta}\chi^\beta, \quad (\eta\gamma^\mu\chi) = \eta^\alpha\epsilon_{\alpha\beta}(\gamma^\mu)^\beta{}_\gamma\chi^\gamma. \tag{A.3}$$

Let $(x^\mu, \theta^\alpha, \overline{\theta}^\alpha)$ be the $\mathcal{N} = 2$ superspace. $\overline{\theta}^\alpha$ is the complex conjugate of the complex spinor θ^α. The complex conjugate of the product of two Grassmann variables α and β is defined by $(\alpha\beta)^* = \beta^*\alpha^*$.

A vector superfield in the Wess-Zumino gauge is expanded as

$$V(v_\mu, \sigma, \lambda, D) = (\theta\gamma^\mu\overline{\theta})v_\mu + i(\theta\overline{\theta})\sigma + \theta^2(\overline{\theta\lambda}) + \overline{\theta}^2(\theta\lambda) + \frac{1}{2}\theta^2\overline{\theta}^2 D. \tag{A.4}$$

The transformation laws of component fields in the Wess-Zumino gauge are

$$\delta\sigma = i(\xi\overline{\lambda}) + i(\overline{\xi}\lambda), \tag{A.5}$$

$$\delta v_\mu = (\xi\gamma_\mu\overline{\lambda}) - (\overline{\xi}\gamma_\mu\lambda), \tag{A.6}$$

$$\delta D = i(\xi\gamma^\mu D_\mu\overline{\lambda}) + i(\overline{\xi}\gamma^\mu D_\mu\lambda) + i(\xi[\sigma, \overline{\lambda}]) + i(\overline{\xi}[\sigma, \lambda]), \tag{A.7}$$

$$\delta\lambda = \frac{i}{2}\gamma^{\mu\nu}\xi F_{\mu\nu} + \gamma^\mu\xi D_\mu\sigma + D\xi. \tag{A.8}$$

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The field strength superfield W_α is defined by

$$W_\alpha = -\frac{1}{8}\overline{D}^2(e^{-2V}D_\alpha e^{2V}). \tag{A.9}$$

We expand a chiral superfield as

$$\Phi(\phi,\psi,F) = \phi + \sqrt{2}i\theta\psi + i\theta^2 F + \overline{\theta} \text{ dependent terms}. \tag{A.10}$$

The supersymmetry transformation including the gauge transformation restoreing the Wess-Zumino gauge is

$$\begin{aligned}
\delta\phi &= \sqrt{2}i(\xi\psi), && \text{(A.11)}\\
\delta\psi &= \sqrt{2}\xi F + \sqrt{2}\overline{\xi}\sigma\phi + \sqrt{2}\gamma^\mu\overline{\xi}D_\mu\phi, && \text{(A.12)}\\
\delta F &= \sqrt{2}i(\overline{\xi}\gamma^\mu D_\mu\psi) - \sqrt{2}i(\overline{\xi}\sigma\psi) - 2i(\overline{\xi\lambda})\phi. && \text{(A.13)}
\end{aligned}$$

An $\mathcal{N}=4$ vector multiplet is made of an $\mathcal{N}=2$ vector multiplet V with components (v_μ,σ,λ,D) and an adjoint chiral multiplet Φ with components (ϕ,χ,F_ϕ). In order to make the $R_{\rm YM} = \mathrm{Spin}(4)$ symmetry manifest we form the following $R_{\rm YM}$ multiplets.

$$\lambda^{A\dot{B}} = \begin{pmatrix} \lambda & \overline{\chi} \\ \chi & -\overline{\lambda} \end{pmatrix}, \quad \phi^{\dot{A}}{}_{\dot{B}} = \begin{pmatrix} \sigma & \sqrt{2}\phi \\ \sqrt{2}\overline{\phi} & -\sigma \end{pmatrix}, \quad F^A{}_B = \begin{pmatrix} D' & \sqrt{2}F_\phi \\ \sqrt{2}\overline{F}_\phi & -D' \end{pmatrix}, \tag{A.14}$$

where D' is the shifted auxiliary field

$$D' = D - [\phi,\overline{\phi}]. \tag{A.15}$$

A hypermultiplet is made of two chiral multiplets $Q(q,\psi,F)$ and $\widetilde{Q}(\widetilde{q},\widetilde{\psi},\widetilde{F})$. These two chiral multiplets must belong to conjugate representations of gauge group to each other. We define the following $R_{\rm YM}$ doublets.

$$q^A = (q^1,q^2) = (q,\overline{\widetilde{q}}), \quad \psi_{\dot{A}} = (\psi_{\dot{1}},\psi_{\dot{2}}) = (\psi,\overline{\widetilde{\psi}}). \tag{A.16}$$

B. Current multiplets

The components of current multiplets are defined by the differentiation of the action $S_{\rm hyper}$ given in (2.5) with respect to the components of vector multiplets.

$$\begin{aligned}
\delta S^I_{\rm hyper} &= -\delta F_I^A{}_B\mu_I^B{}_A - i\delta\lambda_{IA\dot{B}}j_I^{A\dot{B}} + \delta v_{I\mu}J_I^\mu + \delta\phi_I^{\dot{A}}{}_{\dot{B}}K_I^{\dot{B}}{}_{\dot{A}} \\
&\quad +\delta F_{I+1}^A{}_B\widetilde{\mu}_I^B{}_A + i\delta\lambda_{I+1A\dot{B}}\widetilde{j}_I^{A\dot{B}} - \delta v_{I+1\mu}\widetilde{J}_I^\mu - \delta\phi_{I+1}^{\dot{A}}{}_{\dot{B}}\widetilde{K}_I^{\dot{B}}{}_{\dot{A}},
\end{aligned} \tag{B.1}$$

where $S^I_{\rm hyper}$ is the part of $S_{\rm hyper}$ including (q_I,ψ_I).

μ, $\widetilde{\mu}$, j, and $\widetilde{j}$ have been already given in (2.7) and (2.8). The other components are

$$\begin{aligned}
J_I^\mu &= iq_I^A D_\mu\overline{q}_{IA} - iD_\mu q_I^A\overline{q}_{IA} + (\psi_{I\dot{A}}\gamma_\mu\overline{\psi}_I^{\dot{A}}), && \text{(B.2)}\\
\widetilde{J}_I^\mu &= -i\overline{q}_{IA}D^\mu q_I^A + iD^\mu\overline{q}_{IA}q_I^A - (\overline{\psi}_I^{\dot{A}}\gamma^\mu\psi_{I\dot{A}}), && \text{(B.3)}
\end{aligned}$$

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$$K_I{}^{\dot{A}}{}_{\dot{B}} = i\psi_{I\dot{B}}\overline{\psi}_I^{\dot{A}} - \frac{i}{2}\delta^{\dot{A}}_{\dot{B}}\psi_{I\dot{C}}\overline{\psi}_I^{\dot{C}} - \frac{1}{2}\nu_I{}^C{}_C\phi_I{}^{\dot{A}}{}_{\dot{B}} - \frac{1}{2}\phi_I{}^{\dot{A}}{}_{\dot{B}}\nu_I{}^C{}_C + q_I^C\phi^{\dot{A}}_{I+1\dot{B}}\overline{q}_{IC}, \tag{B.4}$$

$$\widetilde{K}_I{}^{\dot{A}}{}_{\dot{B}} = +i\overline{\psi}_I^{\dot{A}}\psi_{I\dot{B}} - \frac{i}{2}\delta^{\dot{A}}_{\dot{B}}\overline{\psi}_I^{\dot{C}}\psi_{I\dot{C}} + \frac{1}{2}\widetilde{\nu}_I{}^C{}_C\phi^{\dot{A}}_{I+1\dot{B}} + \frac{1}{2}\phi^{\dot{A}}_{I+1\dot{B}}\widetilde{\nu}_I{}^C{}_C - \overline{q}_{IC}\phi_I{}^{\dot{A}}{}_{\dot{B}}q_I^C. \tag{B.5}$$

The $\mathcal{N}=4_{\rm YM}$ supersymmetry transformation of μ, $\widetilde{\mu}$, j, and $\widetilde{j}$ are

$$\delta\mu_I{}^A{}_B = i\xi_{B\dot{C}}j_I^{A\dot{C}} - \frac{i}{2}\delta^A_B\xi_{D\dot{C}}j_I^{D\dot{C}}, \tag{B.6}$$

$$\delta\widetilde{\mu}_I{}^A{}_B = i\xi_{B\dot{C}}\widetilde{j}_I^{A\dot{C}} - \frac{i}{2}\delta^A_B\xi_{D\dot{C}}\widetilde{j}_I^{D\dot{C}}, \tag{B.7}$$

$$\delta j_I^{A\dot{B}} = -i\gamma_\mu\xi^{A\dot{B}}J_I^\mu + 2\gamma_\mu\xi^{C\dot{B}}D^\mu\mu_I{}^A{}_C - 2\xi^{A\dot{C}}K_I{}^{\dot{B}}{}_{\dot{C}} + 2\xi^{C\dot{D}}[\mu_I{}^A{}_C, \phi_I{}^{\dot{B}}{}_{\dot{D}}], \tag{B.8}$$

$$\delta\widetilde{j}_I^{A\dot{B}} = -i\gamma^\mu\xi^{A\dot{B}}\widetilde{J}_{I\mu} + 2\gamma^\mu\xi^{C\dot{B}}D_\mu\widetilde{\mu}_I{}^A{}_C - 2\xi^{A\dot{C}}\widetilde{K}_I{}^{\dot{B}}{}_{\dot{C}} + 2\xi^{C\dot{D}}[\widetilde{\mu}_I{}^A{}_C, \phi^{\dot{B}}_{I+1\dot{D}}], \tag{B.9}$$

$$\begin{aligned}\delta J_I^\mu &= \xi_{A\dot{B}}\gamma^{\mu\nu}D_\nu j_I^{A\dot{B}} - \sqrt{2}\xi_{A\dot{B}}\gamma^\mu q_I^A\overline{\Psi}_I^{\dot{B}} + \sqrt{2}\xi^{A\dot{B}}\gamma^\mu\Psi_{I\dot{B}}\overline{q}_{IA} \\ &\quad -[\xi_{B\dot{A}}\gamma^\mu j_I^{B\dot{C}}, \phi_I{}^{\dot{A}}{}_{\dot{C}}] + 2[\xi_{C\dot{B}}\gamma^\mu\lambda_I^{A\dot{B}}, \mu_I{}^C{}_A],\end{aligned} \tag{B.10}$$

$$\begin{aligned}\delta\widetilde{J}_I^\mu &= \xi_{A\dot{B}}\gamma^{\mu\nu}D_\nu\widetilde{j}_I^{A\dot{B}} - \sqrt{2}\xi_{A\dot{B}}\gamma^\mu\overline{\Psi}_I^{\dot{B}}q_I^A + \sqrt{2}\xi^{A\dot{B}}\gamma^\mu\overline{q}_{IA}\Psi_{I\dot{B}} \\ &\quad -[\xi_{C\dot{B}}\gamma^\mu\widetilde{j}_I^{C\dot{A}}, \phi^{\dot{B}}_{I+1\dot{A}}] + 2[\xi_{A\dot{B}}\gamma^\mu\lambda_{I+1}^{C\dot{B}}, \widetilde{\mu}_I{}^A{}_C],\end{aligned} \tag{B.11}$$

$$\begin{aligned}\delta K_I{}^{\dot{A}}{}_{\dot{B}} &= -i\xi_{C\dot{B}}\gamma^\mu D_\mu j_I^{C\dot{A}} + \sqrt{2}i\xi_{C\dot{B}}q_I^C\overline{\Psi}_I^{\dot{A}} + \sqrt{2}i\xi^{C\dot{A}}\Psi_{I\dot{B}}\overline{q}_{IC} \\ &\quad -i[\xi_{D\dot{C}}j_I^{D\dot{A}} - \mathrm{tr}, \phi_I{}^{\dot{C}}{}_{\dot{B}}] - 2i[\xi_{D\dot{B}}\lambda_I^{C\dot{A}}, \mu_I{}^D{}_C] - \mathrm{tr},\end{aligned} \tag{B.12}$$

$$\begin{aligned}\delta\widetilde{K}_I{}^{\dot{A}}{}_{\dot{B}} &= -i\xi_{C\dot{B}}\gamma^\mu D_\mu\widetilde{j}_I^{C\dot{A}} + \sqrt{2}i\xi_{C\dot{B}}\overline{\Psi}_I^{\dot{A}}q_I^C + \sqrt{2}i\xi^{C\dot{A}}\overline{q}_{IC}\Psi_{I\dot{B}} \\ &\quad -i[\xi_{D\dot{C}}\widetilde{j}_I^{D\dot{A}} - \mathrm{tr}, \phi^{\dot{C}}_{I+1\dot{B}}] - 2i[\xi_{C\dot{B}}\lambda_{I+1}^{D\dot{A}}, \widetilde{\mu}_I{}^C{}_D] - \mathrm{tr}.\end{aligned} \tag{B.13}$$

These components are transformed among them linearly up to the equation of motion of ψ_{IA} given in (3.14).

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